The impact of the new Earth gravity models on the measurement of the Lense–Thirring effect

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Abstract

We examine how the new forthcoming Earth gravity models from the CHAMP and, especially, GRACE missions could improve the measurement of the general relativistic Lense–Thirring effect according to the various kinds of observables which could be adopted. In a very preliminary way, we use the first recently released EIGEN2 CHAMP–only and GGM01C GRACE–based Earth gravity models in order to assess the impact of the mismodelling in the even zonal harmonic coefficients of geopotential which represents one of the major sources of systematic errors in this kind of measurement. However, discretion is advised on evaluating the reliability of these results because the Earth gravity models used here, especially EIGEN2, are still very preliminary and more extensive calibration tests must be performed. According to the GGM01C model, the systematic error due to the unmodelled even zonal harmonics of geopotential amounts to 2% for the combination of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II used up to now by Ciufolini and coworkers in the currently performed LAGEOS-LAGEOS II Lense-Thirring experiment, and to 14% for a combination explicitly presented here which involves the nodes only of LAGEOS and LAGEOS II.

Keywords: Lense-Thirring effect, LAGEOS satellites, New Earth gravity models

1 Introduction

An interesting class of Post–Newtonian features is represented by the orbital effects of order $\mathcal{O}(c^{-2})$ induced by the linearized general relativistic gravitoelectromagnetic forces on the motion of a test body freely falling in the gravitational field of a central mass.

Among them, of great interest is the gravitomagnetic Lense–Thirring effect or dragging of inertial frames [1, 2] whose source is the proper angular momentum $\boldsymbol{J}$ of the central mass which acts as source of the gravitational field. Its effect on the precessional motion of the spins $\boldsymbol{s}$ of four freely orbiting superconducting gyroscopes should be tested, among other things, by the important GP–B mission [3] at a claimed accuracy level of the order of 1% or better.

Another possible way to measure such elusive relativistic effects is the analysis of the laser–ranged data of some existing, or proposed, geodetic satellites of LAGEOS–type as LAGEOS, LAGEOS II [4] and the proposed LAGEOS III–LARES [5, 6, 7]. In this case the whole orbit of the satellite is to be thought of as a giant gyroscope whose longitude of the ascending node Ω and the argument of perigee ω (In the original paper by Lense and Thirring the longitude of the pericentre $\varpi = \Omega + \omega$ is used instead of ω) undergo the Lense–Thirring precessions

$$\dot{\Omega}_{\rm LT} = \frac{2GJ}{c^2 a^3 (1-e^2)^{\frac{3}{2}}}, \tag{1}$$

$$\dot{\omega}_{\rm LT} = -\frac{6GJ\cos i}{c^2 a^3 (1-e^2)^{\frac{3}{2}}}, \tag{2}$$

where a, e and i are the semimajor axis, the eccentricity and the inclination, respectively, of the orbit and G is the Newtonian gravitational constant. In recent years first attempts would have yielded a measurement of the Lense–Thirring dragging of the orbits of the existing LAGEOS and LAGEOS II at a claimed accuracy of the order of $20\% - 30\%$ [8, 9]. However, at present, there are some scientists who propose different error budgets [10].

2 The sources of error in the performed test

The observable used in the tests reported in [8, 9] is the following linear combination of the orbital residuals of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II [4]

$$\delta\dot{\Omega}^{\rm LAGEOS} + c_1 \delta\dot{\Omega}^{\rm LAGEOS\ II} + c_2 \delta\dot{\omega}^{\rm LAGEOS\ II} \sim \mu_{\rm LT} 60.2, \tag{3}$$

where $c_1 = 0.304$, $c_2 = -0.350$ and $\mu_{\rm LT}$ is the solved–for least square parameter which is 0 in Newtonian mechanics and 1 in General Relativity. The gravitomagnetic signature is a linear trend with a slope of 60.2 milliarcseconds per year (mas yr^{-1} in the following).

The latest, 2002, measurement of the Lense–Thirring effect, obtained by processing the LAGEOS and LAGEOS II data over a time span of almost 8 years with the orbital processor GEODYN II of the Goddard Space Flight Center, yields [9]

$$\mu_{\rm LT} \sim 1 \pm 0.02 \pm \delta\mu_{\rm LT}^{\rm systematic}, \tag{4}$$

where $\delta\mu_{\rm LT}^{\rm systematic}$ accounts for all the possible systematic errors due to the mismodelling in the various competing classical forces of gravitational and non–gravitational origin affecting the motion of the LAGEOS satellites. In [9] $\delta\mu_{\rm LT}^{\rm systematic}$ is estimated to be of the order of 20%–30%.

The main source of gravitational errors is represented by the aliasing classical secular precessions induced on the node and the perigee of a near Earth satellite by the mismodelled even zonal coefficients of the multipolar expansion of Earth gravitational field: indeed, they mimic the genuine relativistic trend[1]. Eq.(3) is designed in order to cancel out the effects of the first two even zonal harmonics of geopotential which induce mismodelled precessions of the same order of magnitude, or even larger, than the gravitomagnetic shifts, according to the Earth gravity model EGM96 [11] (See Table 1). The evaluation of the impact of the remaining uncancelled even zonal harmonics of higher degree on eq.(3) is of the utmost importance. According to a Root–Sum–Square calculation [12] with the full covariance matrix of EGM96 up to degree $l = 20$ it amounts to almost 13%. However, according to the authors of [10], it would not be entirely correct to automatically extend the validity of the covariance matrix of EGM96, which is based on a multi–year average that spans the 1970, 1980 and early 1990 decades, to any particular time span like that, e.g., of the LAGEOS–LAGEOS II analysis which extends from the middle to the end of the 1990 decade. Indeed, there would not be assurance that the errors in the even zonal harmonics of

[1] Another source of error which would plague an attempted measurement of the Lense–Thirring effect with only one orbital element would be the so called Lense–Thirring 'imprint'. It consists of the fact that in the solutions of the various Earth gravity models General Relativity is assumed to be true, so that the recovered J_l are biased by this a priori assumption. Then, any claimed measurement of the gravitomagnetic precessions based, among other things, on such recovered values of the even zonal harmonics would lack in full credibility and reliability. It turns out that such sort of Lense–Thirring 'imprint' is concentrated, at least for the LAGEOS satellites, mainly in the first two–three even zonal harmonics [4] which do affect the single orbital elements.

the geopotential during the time of the LAGEOS–LAGEOS II experiment remained correlated exactly as in the EGM96 covariance matrix, in view of the various secular, seasonal and stochastic variations that we know occur in the terrestrial gravitational field and that have been neglected in the EGM96 solution. Of course, the same would also hold for any particular future time span of some years. If, consequently, the diagonal part only of the covariance matrix of EGM96 is used, the error due to geopotential, calculated in a Root–Sum–Square fashion, i.e. by taking the square root of the sum of the squares of the individual errors induced by the various even zonal harmonics, amounts to almost[2] 45% [12]. A really conservative upper bound of the error due to geopotential is given by the sum of the absolute values of the individual errors for the various even zonal harmonics. For EGM96 it amounts to 83% (See Table 3). Note that in the EGM96 solution (and in the previous Earth gravity models like JGM3) the recovered even zonal harmonics are highly correlated; in fact, it is likely that the optimistic 13% result obtained with the full covariance matrix is due to a lucky correlation between J_6 and J_8 [10]. Then, in this case, the sum of the absolute values of the individual errors should represent a truly realistic estimate of the impact of the misomodelled even zonal harmonics of geopotential.

Another important class of systematic errors is given by the non–gravitational perturbations which affect especially the perigee of LAGEOS II. For this subtle and intricate matter we refer to [15, 16]. The main problem is that it turned out that their interaction with the structure of LAGEOS II changes in time due to unpredictable modifications in the physical properties of the LAGEOS II surface (orbital perturbations of radiative origin, e.g. the solar radiation pressure and the Earth albedo) and in the evolution of the spin dynamics of LAGEOS II (orbital perturbations of thermal origin induced by the interaction of the electromagnetic radiation of solar and terrestrial origin with the physical structure of the satellites, in particular with their corner–cube retroreflectors). Moreover, such tiny but insidious effects were not entirely modelled in the GEODYN II software at the time of the analysis of [8], so that it is not easy to correctly and reliably assess their impact on the total error budget of the measurement performed during that particular time span. According to the evaluations in [16], the systematic error due to

[2]It is interesting to note that, according to the diagonal part only of the covariance matrix of the GRIM5–C1 Earth gravity model [13], the RSS error due to the uncancelled even zonal harmonics amounts to 13.3%. The GRIM5–S1 and GRIM5–C1 models represent the latest solutions based on conventional satellite tracking data of the pre–CHAMP and GRACE era. They are well tested and calibrated with respect to other existing models [14].

the non–gravitational perturbations over a time span of 7 years amounts to almost 28%. However, according to [10], their impact on the measurement of the Lense–Thirring effect with the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II is, in general, quite difficult to be reliably assessed.

So, by adding quadratically the gravitational and non–gravitational errors of[3] [16] we obtain for the systematic uncertainty $\delta\mu_{\rm LT}^{\rm systematic} \sim 30\%$ if we assume a 13% error due to geopotential, and $\delta\mu_{\rm LT}^{\rm systematic} \sim 54\%$ if we assume a 45% error due to geopotential. The sum of the absolute values of the errors due to gepotential added quadratically with the non–gravitational perturbations would yield a total systematic error of $\delta\mu_{\rm LT}^{\rm systematic} \sim 87.6\%$. It must be noted that the latter estimate is rather similar to those released in [10]. Moreover, it should be considered that the perigee of LAGEOS II is also sensitive to the eclipses effect on certain non–gravitational perturbations. Such features are, generally, not accounted for in all such estimates. An attempt can be found in [17] in which the impact of the eclipses on the effect of the direct solar radiation pressure on the LAGEOS–LAGEOS II Lense–Thirring measurement has been evaluated: it should amount to almost 10% over an observational time span of 4 years.

3 The opportunities offered by the new terrestrial gravity models

From the previous considerations it could be argued that, in order to have a rather precise and reliable estimate of the total systematic error in the measurement of the Lense–Thirring effect with the LAGEOS satellites it would be better to reduce the impact of the geopotential in the error budget and/or discard the perigee of LAGEOS II which is very difficult to handle and is a relevant source of uncertainty due to its great sensitivity to many non–gravitational perturbations.

The forthcoming more accurate Earth gravity models from the CHAMP [18] and, especially, GRACE [19] missions, if the great expectations related to the latter will be finally confirmed, will yield an opportunity to realize both these goals, at least to a certain extent. In order to evaluate quantitatively the opportunities offered by the new terrestrial gravity models we have preliminarily used the recently released EIGEN2 gravity model [20]. It

[3]The estimates obtained there are based on levels of accuracy in knowing the non-gravitational forces which do not coincide with those of the force models included in GEODYN when the analysis of [8] was performed.

is a CHAMP-only gravity field model derived from CHAMP GPS satellite–to–satellite and accelerometer data out of the period 2000, July to December and 2002, September to December. Although higher degree and order terms are solved in EIGEN2, the solution has full power only up to about degree/order 40 due to signal attenuation in the satellite's altitude. Higher degree/order terms are solvable applying regularization of the normal equation system. However, in the case of the LAGEOS satellites it does not pose problems because their nodes and perigees are sensitive to just the first five–six even zonal harmonics[4]. It is important to note that for EIGEN2 it is likely that the released sigmas of the even zonal harmonic coefficients, which are the formal errors, are rather optimistic, at least for the low degree even zonal harmonics up to $l = 20$ [20].

In Table 1 we quote the errors in the measurement of the Lense–Thirring effect with single orbital elements of the LAGEOS satellites according to EGM96 up to degree $l = 70$ (See also Table II of [12]). In Table 2 we quote

Table 1: Systematic gravitational errors $\delta\mu_{\rm LT}^{\rm systematic\ even\ zonals}$ in the measurement of the Lense–Thirring effect with the nodes of the LAGEOS satellites and the perigee of LAGEOS II only according to the EGM96 Earth gravity model up to degree $l = 70$. (C) denotes the full covariance matrix while (D) refers to the diagonal part only used in a RSS way. A pessimistic upper bound has been, instead, obtained from the sum of the absolute values of the individual errors (SAV). In the fifth column the impact of the mismodelling in $\dot J_2^{\rm eff}$ over one year, according to [21], is quoted. The effective coefficient $\dot J_2^{\rm eff}$ accounts for the secular variations of the even zonal harmonics (see below).

LT (mas yr^{-1})	percent error (C)	percent error (D)	percent error (SAV)	$\delta(\dot J_2^{\rm eff})$
$\dot\Omega_{\rm LT}^{\rm LAGEOS}$=30.7	50.3%	199%	341%	8%
$\dot\Omega_{\rm LT}^{\rm LAGEOS\ II}$=31.6	108%	220 %	382%	14%
$\dot\omega_{\rm LT}^{\rm LAGEOS\ II}$ =-57.5	93%	242%	449%	5.4%

the errors in the measurement of the Lense–Thirring effect with single orbital elements of the LAGEOS satellites according to EIGEN2 [5] up to degree $l = 70$. It can be noticed that, for EIGEN2, the results obtained with the

[4]This means that the error in the Lense–Thirring measurement due to the even zonal harmonics of geopotential does not change any more if the even zonal harmonic coefficients of degree higher than 10–12 are neglected in the calculation.

[5]The correlation matrix of EIGEN2 is downloadable from http://op.gfz-potsdam.de/champ/results/ in the form of lower triangular matrix.

Table 2: Systematic gravitational errors $\delta\mu_{\rm LT}^{\rm systematic\ even\ zonals}$ in the measurement of the Lense–Thirring effect with the nodes of the LAGEOS satellites and the perigee of LAGEOS II only according to the EIGEN2 Earth gravity model up to degree $l = 70$. (C) denotes the full covariance matrix while (D) refers to the diagonal part only used in a RSS way. A pessimistic upper bound has been, instead, obtained from the sum of the absolute values of the individual errors (SAV). In the fifth column the impact of the mismodelling in $\dot J_2^{\rm eff}$ over one year, according to [21], is quoted. The effective coefficient $\dot J_2^{\rm eff}$ accounts for the secular variations of the even zonal harmonics (see below).

LT (mas yr^{-1})	percent error (C)	percent error (D)	percent error (SAV)	$\delta(\dot J_2^{\rm eff})$
$\dot\Omega_{\rm LT}^{\rm LAGEOS}$=30.7	71.5%	69%	108%	8%
$\dot\Omega_{\rm LT}^{\rm LAGEOS\ II}$=31.6	107%	107%	144%	14%
$\dot\omega_{\rm LT}^{\rm LAGEOS\ II}$ =-57.5	65%	63%	116%	5.4%

variance matrix in a Root–Sum–Square way are much more similar to those obtained with the full covariance matrix than for EGM96; this fact could be explained by noting that the even zonal harmonics are better resolved and uncorrelated in EIGEN2 than in EGM96 for which, instead, some favorable correlations may finally yield the obtained results (See also [10]). The simple sum of the absolute values of the individual errors for the various degrees yields a pessimistic upper bound of the error due to the bad knowledge of geopotential. However, Table 2 clearly shows that the use of single orbital elements of the LAGEOS satellites in order to measure the Lense–Thirring effect is still unfeasible. Moreover, when a single orbital element is analyzed, the effects of the secular variation of the even zonal harmonics have to be considered as well. It turns out that they can be accounted for by an effective time rate [22]

$$\dot J_2^{\rm eff} \sim \dot J_2 + 0.371\dot J_4 + 0.079\dot J_6 + 0.006\dot J_8 - 0.003\dot J_{10}... \tag{5}$$

whose magnitude is of the order of $(-2.6 \pm 0.3) \times 10^{-11}$ yr^{-1}. Its impact on a possible Lense–Thirring measurement is not negligible at all. It has been evaluated, in a conservative way, by doubling the difference between

In it the recovered even zonal harmonics are disentangled to a higher degree than in EGM96, so that a Root–Sum–Square calculation with the variance matrix should be adequate in reliably assessing the systematic error induced by the mismodelled even zonal harmonics of geopotential.

the maximum and minimum values of the adjusted $\dot{J}_2^{\rm eff}$ for the longest arcs of Table 1 in [21], according to an approach followed in [23].

With regard to eq.(3), it turns out that the systematic error due to the even zonal harmonics of the geopotential, according to the full covariance matrix of EIGEN2 up to degree $l = 70$, amounts to 7%, while if the diagonal part[6] only is adopted it becomes 9% (RSS calculation). The sum of the absolute values yields an upper bound of 16% (See Table 3). Of course, even if the LAGEOS and LAGEOS II data had been reprocessed with the EIGEN2 model, the problems posed by the correct evaluation of the impact of the non–gravitational perturbations on the perigee of LAGEOS II would still persist, unless significant improvements in the modeling of the non–gravitational perturbations on the perigee of LAGEOS II will occur.

A possible approach could be the use of linear combinations of orbital residuals of the nodes and the perigees of the other existing geodetic satellites of LAGEOS type like Starlette, Stella, Ajisai, etc., so to cancel out as many even zonal harmonics as possible. In [24, 25], in which the full covariance matrix of EGM96 up to degree $l = 20$ has been used, it turned out that, due to the lower altitude of the other satellites to be employed, they are more sensitive than the LAGEOS satellites to the even zonal harmonics of higher degree of the geoptential and the combinations including their orbital elements are not competitive with those including only the LAGEOS–LAGEOS II elements. The following combination, which includes the node of Ajisai, seemed to yield a slight improvement in the systematic gravitational error

$$\delta\dot\Omega^{\rm LAGEOS}+c_1\delta\dot\Omega^{\rm LAGEOS\ II}+c_2\delta\dot\Omega^{\rm Ajisai}+c_3\delta\dot\omega^{\rm LAGEOS\ II}\sim\mu_{\rm LT}61.2, \quad (6)$$

with $c_1 = 0.443,\ c_2 = -0.0275,\ c_3 = -0.341$ Indeed, according to the full covariance matrix of EGM96 up to degree $l = 70$, it would amount to 10.3%. Note that it turns out that, with the inclusion of Ajisai, the first ten even zonal harmonics have full power in affecting the systematic error due to geopotential in the Lense–Thirring measurement. If the correlations among the even zonals are neglected, the variance matrix of EGM96, used in a RSS calculation, yields a 64.4% error. The sum of the absolute values of the

[6]It should be noted that the correlations represent the state of processing of the about seven months of CHAMP data incorporated in the EIGEN2 solution. No temporal variations in the zonal coefficients were solved for, so no evolution of coefficients and their correlations can be predicted directly from the solution. In future it will be tried to resolve temporal variations from solutions covering different data epochs (P. Schwintzer, private communication). So, a possible conservative approach might consist in using only the diagonal part of the covariance matrix. However, the calibration of EIGEN2 errors should be extensively and exhaustively checked.

individual errors yields an upper bound of 82%. By using the covariance matrix of EIGEN2 up to degree $l = 70$ the systematic gravitational error raises to 13.4% (13.6% with the diagonal part only of the covariance matrix of EIGEN2 up to degree $l = 70$. RSS calculation.). The sum of the absolute values of the individual errors yields an upper bound of 16% (See Table 3). Since the non–gravitational part of the error budget of eq.(6) is almost similar to that of eq.(3), as it can be inferred from the magnitude of the coefficients of eq.(6) and eq.(3) which weigh the various orbital elements, it is obvious that eq.(6) would not represent any substatntial improvement with respect to the LAGEOS–LAGEOS II observable of eq.(3).

3.1 A new nodes–only combination

A different approach could be followed by taking the drastic decision of canceling out only the first even zonal harmonic of geopotential by discarding at all the perigee of LAGEOS II. The hope is that the resulting gravitational error is reasonably small so to get a net gain in the error budget thanks to the fact that the nodes of LAGEOS and LAGEOS II exhibit a very good behavior with respect to the non–gravitational perturbations. Indeed, they are far less sensitive to their tricky features than the perigee of LAGEOS II. Moreover, they can be easily and accurately measured, so that also the formal, statistical error should be reduced. A possible combination is

$$\delta\dot{\Omega}^{\mathrm{LAGEOS}} + c_1 \delta\dot{\Omega}^{\mathrm{LAGEOS\ II}} \sim \mu_{\mathrm{LT}} 48.2, \tag{7}$$

where $c_1 = 0.546$. A similar approach is proposed in [19], although without quantitative details. According to the full covariance matrix of EIGEN2 up to degree $l = 70$, the systematic error due to the even zonal harmonics from $l = 4$ to $l = 70$ amounts to 8.5 mas yr^{-1} yielding a 17.8% percent error, while if the diagonal part only is adopted it becomes 22% (RSS calculation). EGM96 would not allow to adopt eq.(7) because its full covariance matrix up to degree $l = 70$ yields an error of 47.8% while the error according to its diagonal part only amounts even to[7] 104% (RSS calculation: see Table 3). Note also that eq.(7) preserves one of the most important features of the other combinations of orbital residuals: indeed, it allows to cancel out the very insidious 18.6-year tidal perturbation which is a $l = 2,\ m = 0$ constituent with a period of 18.6 years due to the Moon's node and nominal amplitudes of the order of 10^3 mas on the nodes of LAGEOS and LAGEOS

[7]It reduces to 60% according to the diagonal part only of the covariance matrix of the GRIM5–C1 model (RSS calculation).

II [26]. Moreover, also the secular variations of the even zonal harmonic coefficients of geopotential do not affect the proposed combination: indeed, eq.(7) is designed in order to cancel out just all the effects of the first even zonal harmonic coefficient. On the other hand, the impact of the non–gravitational perturbations on eq.(7) over a time span of, say, 7 years can be quantified in just 0.1 mas yr^{-1}, yielding a 0.3% percent error. The results of Tables 2 and 3 of [6] have been applied to eq.(7) by adding in quadrature the various mismodelled perturbing effects for such combination of orbital elements. To them a 20% mismodelling in the Yarkovsky–Rubincam and Yarkovsky–Schach effects and Earth's albedo and a 0.5% mismodelling in the direct solar radiation pressure have been applied. It is also important to notice that, thanks to the fact that the periods of many gravitational and non–gravitational time–dependent perturbations acting on the nodes of the LAGEOS satellites are rather short, a reanalysis of the LAGEOS and LAGEOS II data over just a few years could be performed. So, with a little time–consuming reanalysis of the nodes only of the existing LAGEOS and LAGEOS II satellites with the EIGEN2 data it would at once be possible to obtain a more accurate and reliable measurement of the Lense–Thirring effect, avoiding the problem of the uncertainties related to the use of the perigee of LAGEOS II. Moreover, it should be noted that the forthcoming, more accurate and robust solutions of the terrestrial gravity fields including the data from both CHAMP and GRACE should yield better results for the systematic error due to the geopotential. Of course, in order to push the gravitational error at the level of a few percent a new LAGEOS–like satellite as the proposed LARES should at least be used [6, 7].

Table 3: Systematic gravitational errors $\delta\mu_{\mathrm{LT}}^{\mathrm{systematic\ even\ zonals}}$ of various combinations of orbital residuals according to EGM96 and EIGEN2 Earth gravity models up to degree $l = 70$. Ob. refers to the combination of orbital residuals adopted. $\mathcal{C}$ refers to the Ciufolini's combination of eq.(3), $\mathcal{A}$ refers to the combination of eq.(6) which includes the node of Ajisai and $\mathcal{I}$ refers to the nodes–only combination of eq.(7) presented here. (C) denotes the use of the full covariance matrix while (D) refers to the diagonal part only (RSS calculation). (SAV) denotes the upper bound obtained from the sum of the absolute values of the individual errors.

Ob.	EGM96 (C)	EGM96 (D)	EGM96 (SAV)	EIG2 (C)	EIG2 (D)	EIG2 (SAV)
$\mathcal{C}$	12.9%	45%	83%	7%	9%	16%
$\mathcal{A}$	10.3%	64.4%	152%	13.4%	12.8%	31.8%
$\mathcal{I}$	48%	104%	177%	17.8%	22%	37%

3.2 First promising results from GRACE

Very recently the first preliminary Earth gravity models including some data from GRACE have been released; among them the GGM01C model[8], which combines the Center for Space Research (CSR) TEG-4 model[9] [27] with data from GRACE, seems to be very promising for our purposes. Indeed, the released sigmas are not the mere formal errors but are approximately calibrated. See Table 4 for the effect on the single elements; the improvements with respect to Table 1 and Table 2 are evident, although not yet sufficiently good in order to allow for a rather accurate measurement of the Lense–Thirring effect by means of only one orbital element.

The error due to geopotential in the combination of eq.(3), evaluated by using the variance matrix only in a Root–Sum–Square fashion, amounts to 2.2% (with an upper bound of 3.1% obtained from the sum of the absolute values of the individual terms). Instead, the combination of eq.(7) would be affected at almost 14% level (RSS calculation), with an upper bound of almost 18% from the sum of the absolute values of the single errors. According also to GGM01C, the combination of eq.(6) seems to be not particularly competitive with respect to that of eq.(3). Indeed, the RSS error amounts to 0.8%, while the upper bound due to the sum of the absolute values of the individual errors is of the order of 2%. See Table 4 also for the combinations of orbital elements. Note that also for GGM01C the covariance matrix is almost diagonal, so that the Root–Sum–Square calculations should yield a realistic evaluation of the systematic error due to the even zonal harmonics of geopotential. It may be interesting to consider the following combination

$$\delta\dot\Omega^{\rm LAGEOS}+c_1\delta\dot\Omega^{\rm LAGEOS\ II}+c_2\delta\dot\Omega^{\rm Ajisai}+c_3\delta\dot\Omega^{\rm Starlette}+c_4\delta\dot\Omega^{\rm Stella}\sim\mu_{\rm LT}57.4, \quad (8)$$

with $c_1 = 4.174$, $c_2 = -2.705$, $c_3 = 1.508$, $c_4 = -0.048$. According to a Root–Sum–Square calculation with the variance matrix of GGM01C up to[10] $l = 42$ the impact of the remaining even zonal harmonics of degree $l \geq 10$ amounts to 21.6 mas yr^{-1} which yields a 37.6% percent error in the

[8] It can be retrieved on the WEB at http://www.csr.utexas.edu/grace/gravity/

[9] The GRACE–only GGM01S model was combined with the TEG-4 information equations (created from historical multi–satellite tracking data; surface gravity data and altimetric sea surface heights) to produce the preliminary gravity model GGM01C.

[10] It has been checked that the error due to the even zonal harmonics remains stable if other even zonal harmonics are added to the calculation. Moreover, it turns also out that, up to $l = 42$ there are no appreciable fluctuations in the calculated classical secular precessions.

Table 4: Systematic gravitational errors $\delta\mu_{\rm LT}^{\rm systematic\ even\ zonals}$ in the measurement of the Lense–Thirring effect with the nodes of the LAGEOS satellites and the perigee of LAGEOS II only and with some combinations according to the GGM01C Earth gravity model up to degree $l = 70$. The sigmas of the even zonal coefficients of this solution are not the mere formal errors but are approximately calibrated. $\mathcal{C}$ refers to the Ciufolini's combination of eq.(3), $\mathcal{A}$ refers to the combination of eq.(6) which includes the node of Ajisai and $\mathcal{I}$ refers to the nodes–only combination of eq.(7) presented here. $\mathcal{M}$ refers to the multi-satellite combination of eq.(8). (D) refers to the diagonal part only used in a RSS way. A pessimistic upper bound has been, instead, obtained from the sum of the absolute values of the individual errors (SAV). In the fourth column the impact of the mismodelling in $\dot J_2^{\rm eff}$ over one year, according to [21], is quoted.

LT (mas yr^{-1})	percent error (D)	percent error (SAV)	$\delta(\dot J_2^{\rm eff})$
$\dot\Omega_{\rm LT}^{\rm LAGEOS}$=30.7	44%	66%	8%
$\dot\Omega_{\rm LT}^{\rm LAGEOS\ II}$=31.6	64%	79%	14%
$\dot\omega_{\rm LT}^{\rm LAGEOS\ II}$ =-57.5	43%	65%	5.4%
$\mathcal{C} = 60.2$	2%	3%	-
$\mathcal{A} = 61.2$	0.8%	1.8%	-
$\mathcal{I} = 48.2$	14%	18%	-
$\mathcal{M} = 57.4$	37%	123%	-

measurement of the Lense–Thirring effect with eq.(8). The upper bound due to the sum of the absolute values of the individual errors amounts to 123%. If the future GRACE–based gravity solutions will improve the high degree ($J_{10},\ J_{12},\ J_{14}, ...$) even zonal harmonics more than the low degree ($J_2,\ J_4,\ J_6,\ J_8$) ones, the combination of eq.(8) could deserve some interest in alternative to that of eq.(7).

4 Conclusions

In this paper we have used, in a very preliminary way, the data from the recently released EIGEN2 Earth gravity model, including six months of CHAMP data, in order to reassess the systematic error due to the even zonal harmonics of the geopotential in the LAGEOS–LAGEOS II Lense–Thirring experiment. The main results are summarized in Table 3 for EGM96 and EIGEN2 and Table 4 for GGM01C which includes the first data from GRACE. It turned out that, by neglecting the correlations between the

various harmonics, such kind of error changes from 45% (or, perhaps more realistically, 83%) of EGM96 to 9% of EIGEN2. Since the correct evaluation of the error budget of such experiment is plagued by the uncertainties due to the impact of the non–gravitational perturbations on the perigee of LAGEOS II, we have considered an observable including only the nodes of LAGEOS and LAGEOS II. It turns out that the systematic error due to the even zonal harmonics of the geopotential, according to EIGEN2 and neglecting the correlations between the various harmonics, amounts to 22%. However, such an observable is almost insensitive to the non–gravitational perturbations which would enter the error budget at a level lower than 1%.

It must be emphasized that the EIGEN2 solution is very preliminary and exhaustive tests should be conducted in order to assess reliably the calibration of the claimed errors, especially in the lower degree even zonal harmonics to which the orbits of the LAGEOS satellites are particularly sensitive. If and when more robust and confident solutions for the terrestrial gravitational field will be hopefully available, especially from GRACE, the proposed observable based on the nodes of the two LAGEOS satellites only could represent a good opportunity for measuring the Lense–Thirring effect in an efficient, fast and reliable way. The first results obtained with the very preliminary GGM01C model, which includes the first data from GRACE and for which the tentatively calibrated errors are available, point toward this direction.

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